An Open-Economy New Keynesian Model[1]

Basic setup. Two open economies produce differentiated *nontraded intermediate goods* indexed by [0,1] in both countries. Each producer of each intermediate goods use all available varieties of domestic labor (and domestic labor only), indexed by [0,1], which interval also indexes the set of domestic households – i.e., each household supplies a distinctive labor variety and has monopoly power over its price (wage).

Final output, which is what households actually consume, is traded between countries and is produced out of locally available intermediates according to the production technology

$$Y = \left[\int_0^1 Y(j)^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}}$$

in Home, and in Foreign,

$$Y^* = \left[\int_0^1 Y^*(j)^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}} .$$

As usual, equilibrium requires that $\xi > 1$.

Home (Foreign) intermediate varieties have local money prices $P_H(j)$ $[P_F^*(j)]$. The reason for assuming that output is made out of nontradable intermediates is so that we can conveniently define the domestic GDP deflator (the price of domestic output in general) on the basis of their prices, in a setting where different varieties may have divergent prices due to price

[1] By Maurice Obstfeld, following Richard H. Clarida, "Reflections on Monetary Policy in the Open Economy," *NBER International Seminar on Macroeconomics*, 2009.

stickiness. Assuming perfect competition in producing final goods (so that price equals cost), the price of Home GDP is

$$P_H = \left[\int_0^1 P_H(j)^{1-\xi} dj \right]^{\frac{1}{1-\xi}},$$

with a parallel formula for the Foreign GDP deflator (in terms of Foreign money), P_F^*.

We assume that a representative household $i \in [0,1]$ maximizes the expected value

$$\mathbb{E}_0 \sum_{t=0}^{\infty} e^{-\delta t} \{u[C_t(i)] - v[N_t(i)]\} = \mathbb{E}_0 \sum_{t=0}^{\infty} e^{-\delta t} \left[\frac{C_t(i)^{1-\sigma}}{1-\sigma} - \frac{N_t(i)^{1+\phi}}{1+\phi} \right], \quad (1)$$

where we define the composite consumption index C in terms of consumption of Home and Foreign final goods:

$$C \equiv 2C_H^{1/2} C_F^{1/2}.$$

Assuming the law of one price holds for tradable final output, $P_F = EP_F^*$ and $P_H^* = P_H/E$, where E is the domestic-currency price of foreign exchange. The domestic CPI is then $P = P_H^{1/2} P_F^{1/2}$ and purchasing power parity holds ($P = EP^*$) – this latter fact being an empirical drawback of this particular model, not an advantage! We define the *terms of trade* as the price of imports in terms of exports,

$$S \equiv P_F/P_H.$$

For firm $j \in [0,1]$ the production function is

$$Y(j) = AN(j),$$

where A is an economy-wide shock $N(j)$ is a CES composite of all the varieties of labor, so that at time t,

$$N_t(j) = \left[\int_0^1 N_t(j,i)^{\frac{\eta_t - 1}{\eta_t}} di\right]^{\frac{\eta_t}{\eta_t - 1}}. \tag{2}$$

Here, the substitution elasticity η_t is time-varying so as to introduce a shock in the markup of the wage over marginal cost of supplying labor. Given the preceding production function, each intermediate firm faces a total wage cost (per unit of the index $N_t(j)$) given by

$$W_t = \left[\int_0^1 W_t(i)^{1-\eta_t} di\right]^{\frac{1}{1-\eta_t}},$$

where $W(i)$ is the nominal wage that household i charges for its variety of labor.

A final useful definition is of *aggregate* "real" marginal cost, which is marginal production cost in terms of the GDP deflator for Home (deflator for Foreign goods in Foreign). If τ is a wage subsidy from the government, this concept is given by

$$MC_t = \frac{(1-\tau_t)W_t}{A_t P_{H,t}} = \frac{(1-\tau_t)P_t W_t}{A_t P_t P_{H,t}} = \frac{(1-\tau_t)S_t^{1/2}}{A_t} \cdot \frac{W_t}{P_t}.$$

As you can see, the terms of trade drive a wedge between the (real) product wage W/P_H and the (real) consumption wage W/P.

Optimization by households. Let N_t denote the intermediate producers' aggregate demand for labor. Then by standard reasoning, eq. (2) implies that for each individual variety of labor i, total demand is given by

$$N_t^d(i) = \int N_t^d(j,i)dj = \left[\frac{W_t(i)}{W_t}\right]^{-\eta_t} N_t. \tag{3}$$

Households may hold bonds denominated in Home or Foreign currency (and possibly other assets). If r_t (r_t^*) is the continuously compounded Home (Foreign) rate of interest, and T_t nominal lumpsum transfers from government, the flow budget constraint for the household (suppressing assets other than bonds) is

$$B_{t+1} + E_t B_{t+1}^* = e^{r_t} B_t + e^{r_t^*} E_t B_t^* + W_t(i) N_t(i) - P_t C_t(i) + T_t.$$

Maximizing (1) subject to this last constraint and (3) yields the first-order conditions:

$$\frac{u'[C_t(i)]}{P_t} = e^{r_t - \delta} \mathbb{E}_t \left\{ \frac{u'[C_{t+1}(i)]}{P_{t+1}} \right\}, \tag{4}$$

$$\frac{u'[C_t(i)]}{P_t} = e^{r_t^* - \delta} \mathbb{E}_t \left\{ \frac{E_{t+1}}{E_t} \cdot \frac{u'[C_{t+1}(i)]}{P_{t+1}} \right\}, \tag{5}$$

$$\frac{u'[C_t(i)](1-\eta_t) N_t(i)}{P_t} + \frac{v'\left[N_t(i)\right] \eta_t N_t(i)}{W_t(i)} = 0.$$

This last equation can be expressed, however, as an equality of the real CPI wage (measured in units of marginal utility) to a markup over the disutility of labor effort:

$$u'[C_t(i)] \cdot \frac{W_t(i)}{P_t} = \frac{\eta_t}{\eta_t - 1} \cdot v'\left[N_t(i)\right] = (1 + \mu_t^w) v'\left[N_t(i)\right].$$

Using the specific isoelastic functional forms for utility, this becomes

$$\frac{W_t}{P_t} = (1 + \mu_t^w) N_t^{\phi} C_t^{\sigma},$$

where, due to the symmetry of households, I have suppressed the i index.

Optimization by firms. A producer of intermediates j faces the demand function

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\xi} Y_t \tag{6}$$

on every date t. These producers use Calvo pricing, such that every period, a fraction $1-N$ of intermediate firms receive a "signal" allowing them to readjust their prices. Write the date t pricing kernel for date $t+k$ money payments as $Q_{t,t+k}=e^{-\delta k}\left[u'(C_{t+k})/P_{t+k}\right]/\left[u'(C_t)/P_t\right]$. A firm that is allowed to set a new price $P^0_{H,t}(j)$ on date t will choose it to maximize the expected present value of revenues conditional on that price not changing: it will solve

$$\max_{P^0_{H,t}(j)} \mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k Q_{t,t+k} Y_{t+k}(j)\left[P^0_{H,t}(j)-P_{H,t+k}MC_{t+k}\right]\right\}$$

subject to (6) with $P_{H,t}(j)=P^0_{H,t}(j)$, where "real" marginal cost MC (in terms of Home output) is as defined earlier, and is exogenous to firm j.

The first-order condition for a maximum is

$$\begin{aligned}
&\mathbb{E}_t\left(\sum_{k=0}^{\infty} N^k Q_{t,t+k}\left\{-\xi Y_{t+k}(j)\left[1-\frac{P_{H,t+k}MC_{t+k}}{P^0_{H,t}(j)}\right]+Y_{t+k}(j)\right\}\right)\\
=\ &\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k Q_{t,t+k}Y_{t+k}(j)\left[(1-\xi)+\xi\frac{P_{H,t+k}MC_{t+k}}{P^0_{H,t}(j)}\right]\right\}=0.
\end{aligned}$$

Multiplication by $P^0_{H,t}(j)/(1-\xi)$ yields the more intuitive expression

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k Q_{t,t+k}Y_{t+k}(j)\left[P^0_{H,t}(j)-\frac{\xi}{\xi-1}P_{H,t+k}MC_{t+k}\right]\right\}=0.$$

In a flexible price model, $P^0_{H,t}(j)=\frac{\xi}{\xi-1}P_{H,t}MC_t$ for all t, where $\frac{\xi}{\xi-1}$, which we shall denote by $1+\mu^p$, is the gross price markup over nominal marginal cost. With Calvo pricing, a particular weighted expectation of deviations from the flex-price markup is optimally set equal to zero. Solving for $P^0_{H,t}(j)$ yields

$$P^0_{H,t}(j)=(1+\mu^p)\frac{\mathbb{E}_t\left\{\sum_{k=0}^{\infty}N^kQ_{t,t+k}Y_{t+k}(j)\left[P_{H,t+k}MC_{t+k}\right]\right\}}{\mathbb{E}_t\left\{\sum_{k=0}^{\infty}N^kQ_{t,t+k}Y_{t+k}(j)\right\}},\qquad(7)$$

which shows that new prices equal a markup over an expected weighted average of future marginal costs, with nearer periods, periods of higher demand, and periods of low consumption getting more weight (the last because profits are most valuable when $u'(C)$ is unexpectedly high).

Equilibrium consumption, trade balance, and terms of trade. In equilibrium supply of final goods equals global demand. For Home and Foreign alike, the value of final output equals the sum of Home and Foreign spending:

$$\begin{aligned} P_H Y &= P_H C_H + P_H C_H^* = \tfrac{1}{2} PC + \tfrac{1}{2} PC^*, \\ P_F Y^* &= P_F C_F + P_F C_F^* = \tfrac{1}{2} PC + \tfrac{1}{2} PC^*. \end{aligned}$$

It follows that the equilibrium terms of trade are:

$$S = \frac{P_F}{P_H} = \frac{Y}{Y^*}.$$

Suppose the countries start out with no net international debts and trade is balanced:

$$P_H Y = PC, P_F Y^* = P^* C^*.$$

Then, by substitution,

$$C = \frac{P_H Y}{P} = S^{-1/2} Y = Y^{1/2} Y^{*1/2} = \frac{P_F Y^*}{P} = C^*.$$

Thus, consumptions will always be equalized, as will real national outputs, leaving no room for trade imbalances. It follows that the conjectured pattern of always balanced trade is an equilibrium. Another implication is that, independently of the menu of assets available, the model produces a complete-markets allocation as its equilibrium.

It is useful to express aggregate labor demand as $N = \int_0^1 N(j)dj$ (over firms), or as

$$N = \frac{Y}{A}\int_0^1 \frac{Y(j)}{Y}dj = \frac{Y}{A}\int_0^1 \left[\frac{P(j)}{P_H}\right]^{-\xi} dj = \frac{YV}{A},$$

so that $Y = AN/V$. Here, V rises as firms' prices become more dispersed, so V is a measure of price dispersion that acts as a negative shock to aggregate productivity.

Flexible-price allocation. Under fully flexible prices, each firm's markup over marginal cost is always $1+\mu^p$ and this also equals the inverse of "real" marginal cost. Denoting the flex-price allocation with overbars we have:

$$\overline{MC} = \frac{(1-\tau)W}{A} \cdot \frac{1}{P_H} = \frac{1}{1+\mu^p} = \frac{\xi - 1}{\xi}.$$

We may also calculate flex-price output (for a given value of η and given foreign output Y^*). Since all firms symmetrically set the same price, V (defined above) equals 1 and $\bar{Y} = A\bar{N}$. In turn,

$$\begin{aligned}
\frac{\bar{W}}{\bar{P}} &= (1+\mu^w)\bar{N}^{\phi}\bar{C}^{\sigma} \\
\Leftrightarrow\quad & \frac{\bar{W}}{\bar{P}_H(\bar{Y}/Y^*)^{1/2}} = (1+\mu^w)\left(\frac{\bar{Y}}{A}\right)^{\phi}(\bar{Y}Y^*)^{\sigma/2} \qquad (8)\\
\Leftrightarrow\quad & \frac{A}{(1-\tau)(1+\mu^p)} = (1+\mu^w)\left(\frac{\bar{Y}}{A}\right)^{\phi}(\bar{Y}Y^*)^{\sigma/2}(\bar{Y}/Y^*)^{1/2}.
\end{aligned}$$

Solving for Home flex-price output gives:

$$\bar{Y} = \left[\frac{A^{1+\phi}\left(Y^*\right)^{\frac{1-\sigma}{2}}}{(1-\tau)(1+\mu^p)(1+\mu^w)}\right]^{1/\kappa}, \qquad (9)$$

where $\kappa = \phi + \frac{\sigma+1}{2}$. Notice how Foreign output shocks are transmitted to Home: An increase in Y^* improves Home's terms of trade, raising the *real* *consumption* wage W/P for a given markup. This effect tends to raise labor supply and output. On the other hand, better terms of trade raise real consumption C, reducing the marginal value of income and thereby reducing labor supply and output. The second effect dominates if $\sigma > 1$, which is the typical presumption. [The distinct roles of the two effects are evident on the two sides of eq. (8) above.] In this case there is negative transmission of output shocks in the flexible-price economy.

The other factors that affect $\bar{Y}$ work as you would expect.

Open-economy IS curve and the natural real rate of interest. Return to eq. (4), suppress the household index i for economy of notation, denote $\ln C$ by c (likewise for other variables), and rewrite as:

$$1 = e^{-\delta} e^{r_t} \mathbb{E}_t \left\{ e^{-\sigma(c_{t+1}-c_t)-(p_{t+1}-p_t)} \right\}.$$

If we linearize around a steady state where $\Delta c = \Delta p = 0$, then

$$\mathbb{E}_t \left\{ e^{-\sigma(c_{t+1}-c_t)-(p_{t+1}-p_t)} \right\} \approx 1 - \sigma \mathbb{E}_t \left(c_{t+1} - c_t \right) - \mathbb{E}_t \left(p_{t+1} - p_t \right),$$

so the preceding Euler equation can be written as

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma} \left(r_t - \mathbb{E}_t \pi_{t+1}^{cpi} - \delta \right),$$

where $\pi_{t+1}^{cpi} \equiv p_{t+1} - p_t$.[2]

[2] If we perform the corresponding linearization on Euler equation (5), then the result is

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma} \left[r_t^* + \mathbb{E}_t(e_{t+1} - e_t) - \mathbb{E}_t \pi_{t+1} - \delta \right],$$

from which we can infer uncovered interest parity,

$$r_t = r_t^* + \mathbb{E}_t(e_{t+1} - e_t).$$

Notice, however, that by taking a *first-order* (linear) approximation, we are ignoring the

In the *closed* economy we would now set $c = y$ and call the result the New Keynesian IS curve. But in the open-economy things are not so simple, since c and y are not necessarily the same. However, for this model we have the information that trade is always balanced, so that

$$p + c = p_H + y \Leftrightarrow c = y - \frac{1}{2}s,$$

and therefore substitution into the linearized Euler equation yields

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{2}\mathbb{E}_t \Delta s_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1}^{cpi} - \delta\right).$$

Moreover, if we define $\pi_{t+1} \equiv p_{H,t+1} - p_{H,t}$, then

$$\pi_{t+1}^{cpi} = \pi_{t+1} + \frac{1}{2}\Delta s_{t+1},$$

and so

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - \delta\right) - \frac{\sigma - 1}{2\sigma}\mathbb{E}_t \Delta s.$$

Because $s = y - y^*$, the preceding can be simplified down to the form:

$$y_t = \mathbb{E}_t y_{t+1} + \frac{1 - (1/\sigma)}{1 + (1/\sigma)}\mathbb{E}_t \Delta y_{t+1}^* - \frac{2}{\sigma + 1}\left(r_t - \mathbb{E}_t \pi_{t+1} - \delta\right).$$

The difference $\sigma - 1$ enters this equation too, but for a different reason than in the equation for $\bar{Y}$. Faster output growth abroad means improving Home terms of trade and thus lower CPI inflation, given π. Other things equal, this means an increase in the real rate of interest and lower output, with a semi-elasticity that is inversely proportional to σ. On the other hand (as

influence of potentially important second-order effects, which would remain if instead the approximation were *quadratic*. The omitted variance and covariance terms are risk premia reflecting the effects of uncertainty and risk aversion.

before), improving terms of trade imply that real consumption c is growing faster than output. So for any rate of real consumption growth, improving terms of trade imply lower output growth (with elasticity 1), and hence a higher level of current output given future output.

We can define the *natural real rate of interest* $\overline{rr}$ for the Home country (given exogenous Foreign output) as the real interest rate (expressed in terms of Home output) that would be consistent with the IS curve at the flexible-price output level:

$$\bar{y}_t = \mathbb{E}_t \bar{y}_{t+1} + \frac{1 - (1/\sigma)}{1 + (1/\sigma)} \mathbb{E}_t \Delta y^*_{t+1} - \frac{2}{\sigma + 1} \left(\overline{rr}_t - \delta \right).$$

Solving, we find that

$$\overline{rr}_t = \delta + \frac{\sigma + 1}{2} \mathbb{E}_t \Delta \bar{y}_{t+1} + \frac{\sigma - 1}{2} \mathbb{E}_t \Delta y^*_{t+1}.$$

In a closed economy the corresponding formula would be $\overline{rr}_t = \delta + \sigma \mathbb{E}_t \Delta \bar{y}_{t+1}$ (directly from the Euler equation). However, the formula above corrects for expected terms of trade changes according to

$$\begin{aligned} \overline{rr}_t &= \delta + \sigma \mathbb{E}_t \Delta \bar{y}_{t+1} + \frac{\sigma - 1}{2} \mathbb{E}_t \left(\Delta y^*_{t+1} - \Delta \bar{y}_{t+1} \right) \\ &= \delta + \sigma \mathbb{E}_t \Delta \bar{y}_{t+1} - \frac{\sigma - 1}{2} \mathbb{E}_t \Delta \bar{s}_{t+1}. \end{aligned}$$

Other things being equal, expected losses in the terms of trade ($\mathbb{E} \Delta s > 0$) lower the natural real rate of interest if $\sigma > 1$.

After subtraction, the New Keynesian *IS* curve can be written in terms of the output gap, $\tilde{y} = y - \bar{y}$:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{2}{\sigma + 1} \left(r_t - \mathbb{E}_t \pi_{t+1} - \overline{rr}_t \right).$$

Notice that when $\sigma > 1$, the coefficient on the real interest gap in the IS curve, which equals $\frac{1}{\frac{\sigma}{2}+\frac{1}{2}}$, is *above* the value $1/\sigma$ for a closed economy. This means that the IS curve (with the interest rate on the vertical axis) is flatter in this open economy: changes in the interest rate are associated with bigger changes in output. The basic reason is that a rise in output due to a cut in the interest rate also puts negative pressure on the terms of trade, raising $p_F - p_H$ and thereby reducing the consequent associated increase in c. To compensate, y must rise more than would be the case in a closed economy.

New Keynesian Phillips curve. Define $\tilde{Q}_{t.t+k} \equiv Q_{t,t+k}/\beta^k$, where $\beta \equiv e^{-\delta}$, and rewrite eq. (7) in the following form after dividing numerator and denominator by $\bar{Y}$,and dividing the entire expression by $P_{H,t-1}$:

$$\frac{P^0_{H,t}(j)}{P_{H,t-1}} = (1+\mu^p)\frac{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\tilde{Q}_{t,t+k}\left[\frac{Y_{t+k}(j)}{\bar{Y}}\right]\left[\frac{P_{H,t+k}}{P_{H,t-1}}MC_{t+k}\right]\right\}}{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\tilde{Q}_{t,t+k}\left[\frac{Y_{t+k}(j)}{\bar{Y}}\right]\right\}}.$$

In a flexible-price steady state with zero output or consumption growth, $\tilde{Q} = 1$, $Y_{t+k}(j)/\bar{Y} = 1$, and $1/MC = 1/\overline{MC} = 1 + \mu^p$. The preceding has a convenient log-linear approximation. To derive it, multiply through by $1/\overline{MC}$, use lower-case letters once again to denote natural logs, and re-write the preceding as

$$\begin{aligned}
\frac{P^0_{H,t}(j)}{P_{H,t-1}} &= \frac{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k e^{\tilde{q}+y+\left(p_{H,t+k}-p_{H.t-1}\right)+\left(mc_{t+k}-\overline{mc}\right)}\right\}}{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k e^{\tilde{q}+y}\right\}} \\
&\approx \frac{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\left[1+\tilde{q}+y+\left(p_{H,t+k}-p_{H.t-1}\right)+\left(mc_{t+k}-\overline{mc}\right)\right]\right\}}{\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\left(1+\tilde{q}+y\right)\right\}} \\
&= \frac{1+(1-\beta N)\,\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\left(\tilde{q}+y+p_{H,t+k}-p_{H.t-1}+mc_{t+k}-\overline{mc}\right)\right\}}{1+(1-\beta N)\,\mathbb{E}_t\left\{\sum_{k=0}^{\infty} N^k\beta^k\left(\tilde{q}+y\right)\right\}}.
\end{aligned}$$

Taking natural logs of both sides above and again using the approximation $\ln(1+x) \approx x$, we get the useful relationship

$$p_{H,t}^{0} - p_{H,t-1} \approx (1-\beta N)\,\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} N^k \beta^k \left[(p_{H,t+k} - p_{H.t-1}) + (mc_{t+k} - \overline{mc}) \right] \right\}. \tag{10}$$

This expression is useful because it will eventually allow us to explain the inflation rate for Home products in terms of future expected inflation and real marginal cost. Observe that the aggregate price of Home goods $P_{H,t}$ is given by

$$P_{H,t} = \left[N \left(P_{H,t-1} \right)^{1-\xi} + (1-N) \left(P_{H,t}^{0} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}}.$$

Similar log-linear approximations to those already done lead to the intuitive approximation

$$\pi_t = p_{H,t} - p_{H,t-1} \approx (1-N) \left(p_{H,t}^{0} - p_{H,t-1} \right). \tag{11}$$

By combining this with eq. (10), we find an expression for aggregate inflation – a Phillips curve.

As an intermediate step, observe that

$$\begin{aligned} (1-\beta N) \sum_{k=0}^{\infty} N^k \beta^k \left(p_{H,t+k} - p_{H.t-1} \right) &= (1-\beta N) \sum_{k=0}^{\infty} N^k \beta^k \left(\pi_{t+k} + \pi_{t+k-1} + ... + \pi_t \right) \\ & \quad (1-\beta N) \left(\pi_t \sum_{k=0}^{\infty} N^k \beta^k + \pi_{t+1} \sum_{k=1}^{\infty} N^k \beta^k + ... \right) \\ &= (1-\beta N) \left(\frac{1}{1-\beta N} \pi_t + \frac{\beta N}{1-\beta N} \pi_{t+1} + ... \right) \\ &= \sum_{k=0}^{\infty} N^k \beta^k \pi_{t+k}. \end{aligned}$$

Letting $\widehat{mc} \equiv mc - \overline{mc}$, we therefore deduce that

$$p_{H,t}^{0} - p_{H,t-1} = \mathbb{E}_t \sum_{k=0}^{\infty} N^k \beta^k \pi_{t+k} + (1-\beta N) \mathbb{E}_t \sum_{k=0}^{\infty} N^k \beta^k \widehat{mc}_{t+k}.$$

Now, this last equation is a solution to the difference equation

$$p^0_{H,t} - p_{H,t-1} = \beta N \mathbb{E}_t \left\{ p^0_{H,t+1} - p_{H,t} \right\} + \pi_t + (1-\beta N)\widehat{mc}_t.$$

However, eq. (11) implies that $p^0_{H,t} - p_{H,t-1} = (1-N)^{-1}\pi_t$. Substituting this into the preceding difference equation results in:

$$\pi_t = \beta N \mathbb{E}_t \pi_{t+1} + (1-N)\pi_t + (1-N)(1-\beta N)\widehat{mc}_t$$

or

$$\begin{aligned} \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-N)(1-\beta N)}{N}\widehat{mc}_t \\ &= \beta \mathbb{E}_t \pi_{t+1} + \varphi \widehat{mc}_t, \end{aligned} \tag{12}$$

where $\varphi \equiv (1-N)(1-\beta N)/N$. Superficially, this equation is the same as in the closed-economy case, but the difference for the open economy lies in the different factors affecting real marginal cost. Let's look more closely at that. Recall that real marginal cost can be expressed as

$$\begin{aligned} MC &= \frac{(1-\tau)(Y/Y^*)^{1/2}}{A} \cdot \frac{W}{P} = \frac{(1-\tau)(1+\mu^w)(Y/Y^*)^{1/2}}{A} \cdot C^\sigma N^\phi \\ &= \frac{(1-\tau)(1+\mu^w)(Y/Y^*)^{1/2}}{A} \cdot (YY^*)^{\sigma/2}(VY/A)^\phi \\ &= (1-\tau)(1+\mu^w)A^{-(1+\phi)}Y^\kappa \left(Y^*\right)^{\frac{\sigma-1}{2}} V^\phi. \end{aligned}$$

On the assumption that $V \approx 1$, we may therefore approximate

$$\begin{aligned} \widehat{mc} &\approx \ln(1-\tau) + \ln(1+\mu^w) - (1+\phi)a + \kappa y + \left(\frac{\sigma-1}{2}\right) y^* + \ln(1+\mu^p) \\ &\approx \kappa y - \left[(1+\phi)a - \left(\frac{\sigma-1}{2}\right) y^* - \ln(1-\tau) - \ln(1+\bar{\mu}^w) - \ln(1+\mu^p)\right] + (\mu^w - \bar{\mu}^w) \\ &= \kappa(y-\bar{y}) + (\mu^w - \bar{\mu}^w) = \kappa\tilde{y} + (\mu^w - \bar{\mu}^w). \end{aligned}$$

where you may recall that $\kappa = \phi + \frac{1}{2}(\sigma + 1)$ (which weight would be the larger number $\phi + \sigma$ in the closed economy, if $\sigma > 1$, with a zero weight on y^*).[3] Combination of this with eq. (12) results in the *New Keynesian Phillips curve*,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \tilde{y}_t + u_t,$$

where $\lambda \equiv \varphi\kappa$ and $u_t \equiv \varphi(\mu_t^w - \bar{\mu}^w)$. Because λ is smaller than it would be in a closed-economy for $\sigma > 1$, openness flattens the Phillips curve: a given change in the output gap is associated with lower inflation, other things being equal. Higher output has a smaller impact on marginal cost in this case. There are two factors at work here causing open economies to differ from closed:

1. First, for a given real CPI wage W/P, a rise in Y lowers P_H/P_F, raising the real product wage (with elasticity 1); this effect, by itself, would tend to accentuate the effect of positive of output on marginal cost.

2. However, there is second effect working in the opposite direction: by depressing P_H/P_F, a rise in Y tends to dampen the associated increase in real consumption C, implying a higher $u'(C)$ and therefore a lower product wage (with elasticity σ).

When $\sigma > 1$, the second effect dominates and open-economy marginal cost is less sensitive to the output gap than in a closed economy.

The flattening of the Phillips curve could be bad news for policymakers, since it implies that a bigger negative output gap would have to be engineered

[3]Note also that the flex-price output measure $\bar{y}$ that defines the output gap is specified as a function of the *mean* wage markeup $\bar{\mu}^w$.

to attain a given reduction in inflation. Recall other ways in which openness could complicate monetary policy in this model: though the effect of Foreign output on the "natural" level of output, and therefore on the output gap associated with any given level of domestic output; and through the effect of Foreign output on the natural real rate of interest $\overline{rr}$.

Optimal monetary policy. For simplicity, we analyze policy under *discretion.* That is, we assume the policymaker takes expectations of future inflation as given when choosing inflation today. However, this setup raises the possibility of *dynamic inconsistency* in the mode of Kydland-Prescott, Calvo, and Barro-Gordon. Specifically, there are two sources of dynamic inconsistency in the model, one familiar from the macro literature, one less so. Regarding the first source, because of the markups μ^p and μ^w, output and labor supply are suboptimally low, and the policymaker could push these closer to optimal levels by creating unanticipated inflation and currency depreciation.[4] Regarding the second source, recall the analysis of the optimum tariff in trade theory. In this model, the global demand for domestic final output has an elasticity of 1, which is not internalized by the monopolistic producers of intermediates (who face demand curves with price elasticity $\xi > 1$).[5] This national-level distortion implies an incentive for the policymaker to engineer an unexpected currency *appreciation* so as to reduce output and improve the terms of trade (thereby reducing household labor effort). The labor subsidy

[4]For example, see Obstfeld and Rogoff, *Foundations* (1996), chapter 10, exercise 4.

[5]For discussion, see the appendix to Maurice Obstfeld and Kenneth Rogoff, "Risk and Exchange Rates," in Elhanan Helpman and Efraim Sadka, eds., *Economic Policy in the International Economy: Essays in Honor of Assaf Razin* (Cambridge: Cambridge University Press, 2003).

τ can be set by the fiscal authority so that these two sources of dynamic inconsistency exactly balance out in the steady state, removing any source of trend inflation or deflation in the model. It can be shown that the level of subsidy that does this satisfies

$$\tfrac{1}{2}(1-\tau)(1+\bar{\mu}^w)(1+\mu^p)=1,$$

and we assume that the natural flex-price output level $\bar{Y}$ and hence the output gap are defined based on this subsidy value.[6]

If Home policymakers wish to maximize the utility of the representative household in this framework, then a second-order approximation to the utility function at the policy optimum is

$$\mathcal{W}_t=-\tfrac{1}{2}\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\left(\pi_{t+k}^2+\alpha\tilde{y}_{t+k}^2\right),$$

where $\alpha=\varphi\kappa/\xi=\lambda/\xi$.[7] Note that when $\sigma>1$, an open-economy policymaker will place relatively less weight on output stabilization (λ, the slope coefficient in the Phillips curve, is lower). Intuitively, the relatively slow pass-through of demand pressure to inflation gives the policymaker more leeway to tolerate larger output gaps. On the other hand, a small value of ξ implies a large monopolistic distortion and a bigger relative weight on output stabilization.

Under policy discretion the monetary policymaker minimizes $-\frac{1}{2}\beta^k\left(\pi_t^2+\alpha\tilde{y}_t^2\right)$ subject to

$$\pi_t=\beta\mathbb{E}_t\pi_{t+1}+\lambda\tilde{y}_t+u_t,$$

[6] For a detailed derivation, consult Richard Clarida, Jodi Galí, and Mark Gertler, "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics* 49 (July 2002): 879-904.

[7] See Clarida, Galí, and Gertler, *ibid.*

taking as *given* the expectation $\mathbb{E}_t\pi_{t+1}$. If θ denotes the Lagrange multiplier on this constraint, first-order conditions are

$$\pi_t = \theta, \;\; \alpha\tilde{y}_t = -\theta\lambda,$$

which imply that

$$\tilde{y}_t = -\frac{\theta\lambda}{\alpha} = -\frac{\lambda\pi_t}{\alpha} = -\frac{\lambda\pi_t}{\lambda/\xi} = -\xi\pi_t.$$

For example, if inflation is above target, the output gap should be negative, and more so the bigger is ξ (which varies inversely with the degree of monopolistic distortion).

Given the policy function above, inflation will follow

$$\begin{aligned} \pi_t &= \beta\mathbb{E}_t\pi_{t+1} - \lambda\xi\pi_t + u_t \\ &\Rightarrow \;\; \pi_t = \frac{\beta}{1+\lambda\xi}\mathbb{E}_t\pi_{t+1} + \frac{1}{1+\lambda\xi}u_t. \end{aligned}$$

People understand that the policymaker will decide inflation on future dates in the same manner as today, so we may iterate the preceding relationship forward. Assume that the Phillips curve equation error u_t (which reflects changes in the wage markup) follows and AR(1) with autoregressive parameter ρ. Then the result is

$$\pi_t = \frac{1}{1+\lambda\xi}\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\lambda\xi}\right)^k \rho^k u_t = \frac{u_t}{1+\lambda\xi-\beta\rho} \equiv \psi u_t.$$

Inflation depends on expected future wage shocks, which tend to reduce output but are partially offset by monetary policy. Notice that, therefore,

$$\mathbb{E}_t\pi_{t+1} = \rho\pi_t.$$

How an monetary policy bring about this optimum point on the Phillips output-inflation tradeoff? We need to use the IS curve,

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma_0}\left(r_t - \mathbb{E}_t \pi_{t+1} - \overline{rr}_t\right), \ \sigma_0 \equiv \frac{\sigma+1}{2},$$

to find the appropriate choice of policy nominal interest rate, r_t. Solving,

$$r_t = \overline{rr}_t + \mathbb{E}_t \pi_{t+1} + \sigma_0 \left(\mathbb{E}_t \tilde{y}_{t+1} - \tilde{y}_t\right). \tag{13}$$

Substituting using the policy rule yields

$$\begin{aligned} r_t &= \overline{rr}_t + \mathbb{E}_t \pi_{t+1} + \sigma_0(-\xi \mathbb{E}_t \pi_{t+1} + \frac{\xi}{\rho} \mathbb{E}_t \pi_{t+1}) \\ &= \overline{rr}_t + \left[1 + \sigma_0 \xi \left(\frac{1-\rho}{\rho}\right)\right] \mathbb{E}_t \pi_{t+1}. \end{aligned}$$

This can be looked at as an instance of the *Taylor rule* (though it is very specific to our modeling assumptions). The nominal interest rate increases better than one-for-one in response to expected inflation. Here, with $\sigma > 1$, $\sigma_0 < \sigma$, and so the policy interest rate response is milder in the open-economy case. The reason is that, as we have seen, a given real interest rate change (the latter measured in terms of domestic output) elicits a bigger output response in the open economy.

The nominal exchange rate. Since we are taking Foreign output as exogenous, $s_t = \tilde{y}_t + \bar{y}_t - y_t^*$, and we can invoke eq. (9) for $\bar{Y}$ to write

$$\bar{y}_t = \frac{1}{\kappa}\left[(1+\phi)a_t + \tfrac{1}{2}(1-\sigma)y_t^*\right],$$

given what we have assumed about the setting of τ. Because the law of one price has been assumed for imports, $p_F = e + p_F^*$, and so, by the definition of the terms of trade,

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t - \pi_t^*.$$

Therefore, we have

$$
\begin{aligned}
e_t &= e_{t-1} + \Delta\tilde{y}_t + \frac{1+\phi}{\kappa}\Delta a_t + \frac{(1-\sigma)}{2\kappa}\Delta y_t^* - \Delta y_t^* + \pi_t - \pi_t^* \\
&= e_{t-1} - \xi\psi\Delta u_t - \frac{\phi+\sigma}{\kappa}\Delta y_t^* + \psi u_t - \pi_t^* \\
&= e_{t-1} - \psi(\xi-1)u_t + \psi\xi u_{t-1} + \frac{1+\phi}{\kappa}\Delta a_t - \frac{\phi+\sigma}{\kappa}\Delta y_t^* - \pi_t^*.
\end{aligned}
$$

Some implications:

- In response to cost-push pressure ($u_t > 0$), there is short run appreciation (because output is reduced by interest-rate increase), followed by permanent depreciation to accommodate higher inflation.
- Higher Home productivity calls for currency depreciation (to stabilize output gap and inflation).
- A rise in Foreign output calls for currency appreciation (so that our terms of trade can improve without pushing up domestic inflation, and also to stabilize our output gap).
- Higher Foreign inflation causes our currency to appreciate nominally.
- Due to the discretionary monetary policy framework, the nominal exchange rate has a unit root.